**Simultaneous surds equation**

**1.** Solve $\left\{\begin{matrix}\sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}=\frac{7}{\sqrt{xy}}\\\sqrt[4]{x^{3}y}-\sqrt[4]{xy^{3}}=\sqrt{12}\end{matrix}\right.$ .

 $\left\{\begin{matrix}\sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}=\frac{7}{\sqrt{xy}} …(1)\\\sqrt[4]{x^{3}y}-\sqrt[4]{xy^{3}}=\sqrt{12}…(2)\end{matrix}\right.$

 For $x,y>0$, from (1), $x-y=7 …(3)$

 From (2), $\sqrt[4]{xy}\left(\sqrt{x}-\sqrt{y}\right)=\sqrt{12}$

 Squaring we get $\sqrt{xy}\left(\sqrt{x}-\sqrt{y}\right)^{2}=12 …(4)$

 From (3), $\sqrt{x}-\sqrt{y}=\frac{7}{\sqrt{x}+\sqrt{y}} …(5)$

 $\left(5\right)\downright \left(4\right), \sqrt{xy}\left(\frac{7}{\sqrt{x}+\sqrt{y}}\right)^{2}=12$

 $⟹49\sqrt{xy}=12\left(x+2\sqrt{xy}+y\right)$

 $⟹12x-25\sqrt{xy}+12y=0$

 $⟹12\left(\sqrt{x}\right)^{2}-25\sqrt{xy}+12\left(\sqrt{y}\right)^{2}=0$

 $⟹\left(4\sqrt{x}-3\sqrt{y}\right)\left(3\sqrt{x}-4\sqrt{y}\right)=0$

 $⟹\sqrt{\frac{x}{y}}=\frac{3}{4} or \frac{4}{3}$

 $⟹\frac{x}{y}=\frac{9}{16} or \frac{16}{9}$

 $⟹x=\frac{9}{16}y or x=\frac{16}{9}y …(6)$

 $(6)\downright \left(3\right)$, $\frac{9}{16}y-y=7 or \frac{16}{9}y-y=7 $

 $-\frac{7}{16}y=7 or \frac{7}{9}y=7$

 $y=-16 or y=9$

 $\left(x,y\right)=\left(-9,-16\right) or \left(16,9\right)$.

 Checking with (1) and (2), we have $\left(x,y\right)= \left(16,9\right)$.

 Similarly, if $x,y<0$, $\left(x,y\right)=\left(-16,-9\right)$.

 Hence, $\left(x,y\right)=\left(-16,-9\right) or \left(16,9\right)$.

**2.** Solve $\left\{\begin{matrix}\sqrt{7x+y}+\sqrt{x+y}=6\\\sqrt{x+y}-y+x=2\end{matrix}\right.$ .

 $\left\{\begin{matrix}\sqrt{7x+y}+\sqrt{x+y}=6 …(1)\\\sqrt{x+y}-y+x=2 …(2)\end{matrix}\right.$

 $\left(1\right)-\left(2\right), \sqrt{7x+y}-y+x=x-4⟹\sqrt{7x+y}=4-y+x …(3)$

 From (1),

 $\left(\sqrt{7x+y}+\sqrt{x+y}\right)\left(\sqrt{7x+y}-\sqrt{x+y}\right)=6\left(\sqrt{7x+y}-\sqrt{x+y}\right)$

 $\left(7x+y\right)-\left(x+y\right)=6\left(\sqrt{7x+y}-\sqrt{x+y}\right)$

 $6x=6\left(\sqrt{7x+y}-\sqrt{x+y}\right)$

 $\sqrt{7x+y}-\sqrt{x+y}=x …(4)$

 $\left(4\right)+\left(2\right), \sqrt{7x+y}-y+x=x+2$

 $⟹\sqrt{7x+y}=2+y …(5)$

 $\left(3\right)=\left(5\right), 4-y+x =2+y $

 $⟹x=2y-2 …(6)$

 $\left(6\right)\downright \left(5\right), \sqrt{7\left(2y-2\right)+y}=2+y $

 $⟹\sqrt{15y-14}=2+y $

 $⟹15y-14=4+4y+y^{2}$

 $∴y^{2}-11y-18=0$

 $⟹\left(y-2\right)\left(y-9\right)=0$

 $⟹y=2, 9$

 When $y=9, x=16$ (rejected)

 When $y=2, x=2$ (satisfies (1) and (2))

 Solution : $\left(x,y\right)=(2,2)$.

 Should also check : $\left(6\right)\downright \left(3\right)$. (Result is the same.)

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